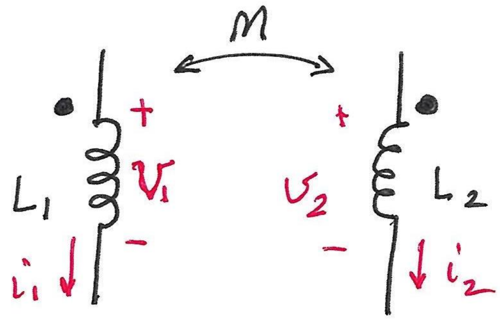
Mutual Inductance



L_1 and L_2 are called self inductance.

M is called mutual inductance

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

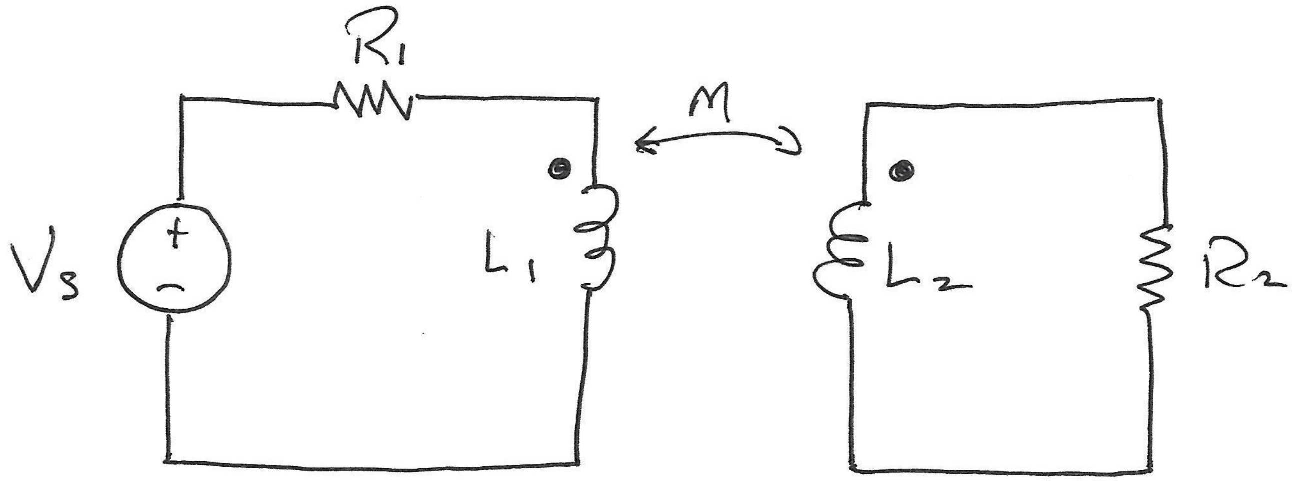
In AC notation:

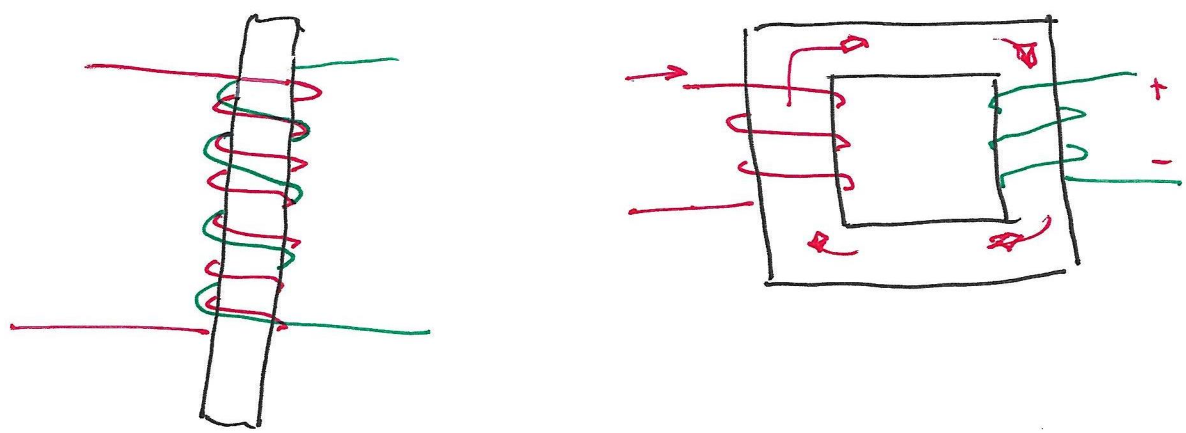
$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

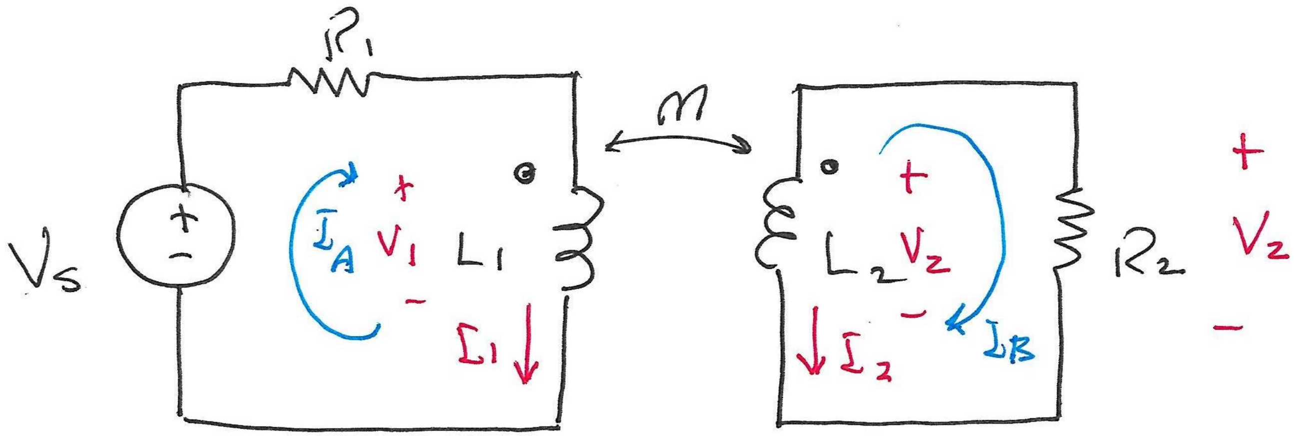
$$V_2 = j\omega M I_1 + j\omega L_2 I_2$$

due to mutual coupling

self-induced


R_1
M
V_s
+
−
L_1
L_2
R_2


+
−



$$V_1 = j\omega L_1 \overset{I_A}{\cancel{I_1}} \overset{-}{\cancel{+}} j\omega M \overset{I_B}{\cancel{I_2}}$$

$$V_2 = j\omega M \overset{I_A}{\cancel{I_1}} \overset{-}{\cancel{+}} j\omega L_2 \overset{I_B}{\cancel{I_2}}$$

$$V_1 = j\omega L_1 I_A - j\omega M I_B$$

$$V_2 = j\omega M I_A - j\omega L_2 I_B$$

For loop A:

$$-V_s + R_1 I_A + V_1 = 0 \qquad (1)$$

For loop B:

$$-V_2 + R_2 I_B = 0 \qquad (2)$$

$$(1) \Rightarrow \quad -V_s + R_1 I_A + j\omega L_1 I_A - j\omega M I_B = 0$$

$$(2) \Rightarrow \quad -j\omega M I_A + j\omega L_2 I_B + R_2 I_B = 0$$

In matrix form:

$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix}$$

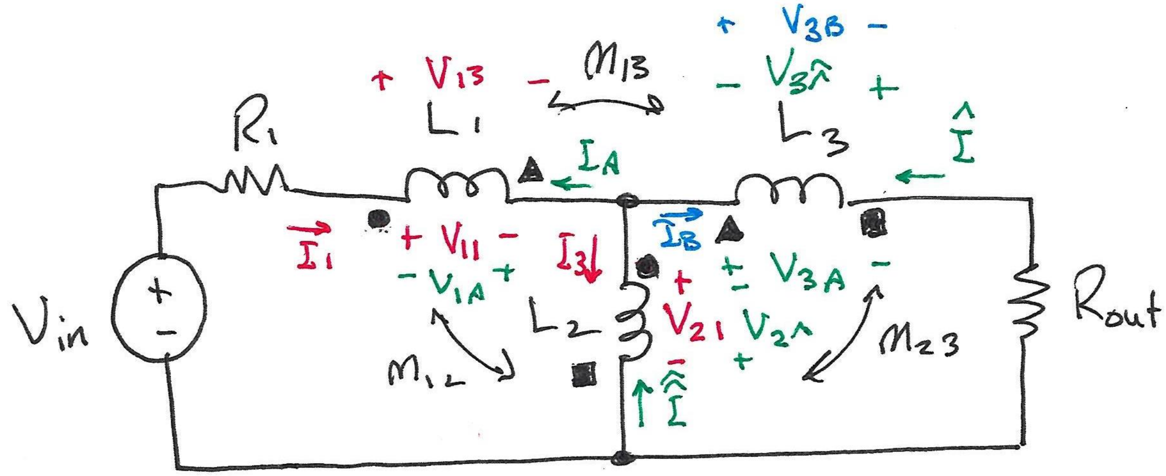
Solve for I_B:

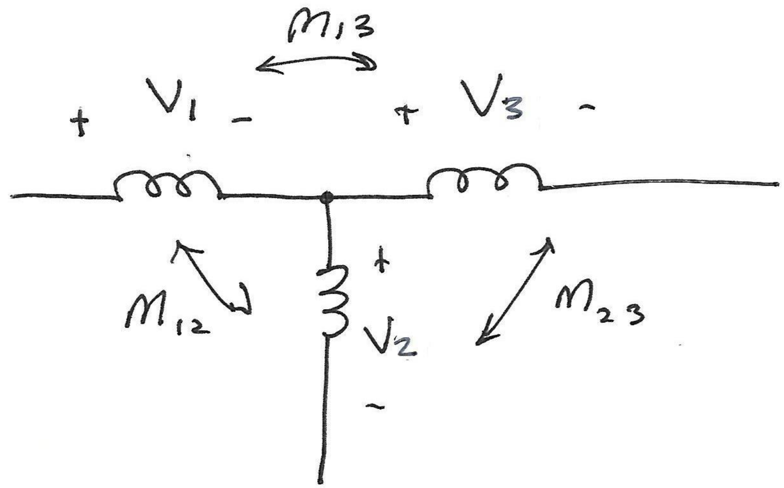
$$I_B = \frac{\begin{vmatrix} R_1 + j\omega L_1 & V_s \\ -j\omega M & 0 \end{vmatrix}}{\begin{vmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega L_2 \end{vmatrix}} = \frac{j\omega M V_s}{(R_1 + j\omega L_1)(R_2 + j\omega L_2) - (-j\omega M)(-j\omega M)}$$

$$= \frac{j\omega M V_s}{R_1 R_2 + j\omega R_1 L_2 + j\omega L_1 R_2 - \omega^2 L_1 L_2 + \omega^2 M}$$

$$= \frac{j\omega M V_s}{(R_1 R_2 - \omega^2 L_1 L_2 + \omega^2 M^2) + j\omega (R_1 L_2 + L_1 R}$$

Then $V_2 = R_2 I_B$

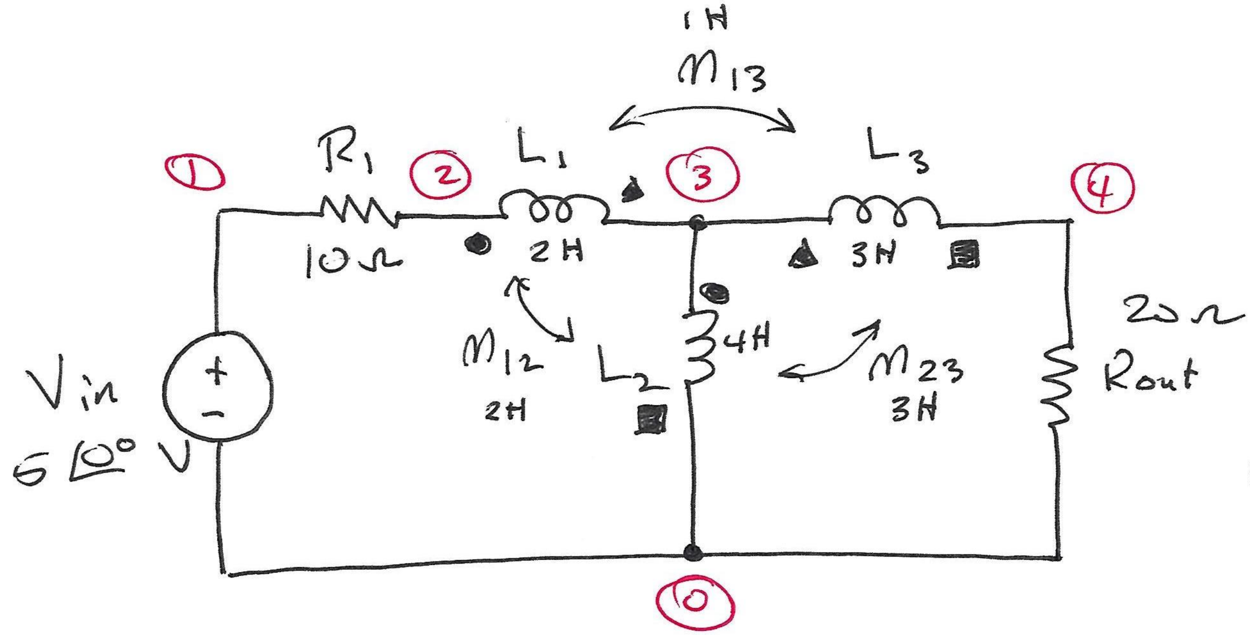




$$V_1 = V_{13} + V_{11} - V_{1A}$$

$$V_3 = V_{3B} - V_{3\wedge} + V_{3A}$$

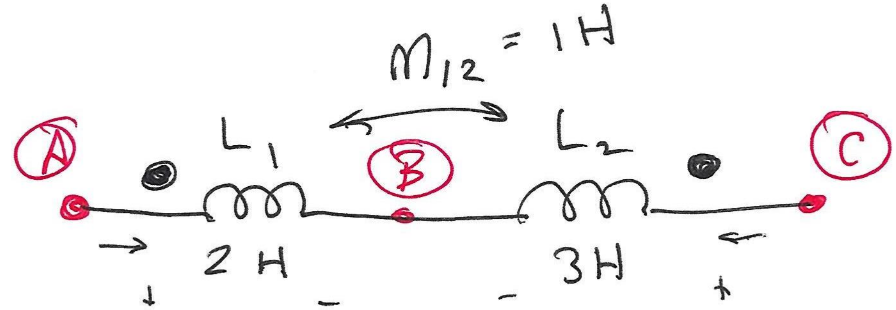
$$V_2 = V_{21} - V_{2\wedge} + V_{23}$$



$$k_{12} = \frac{2}{\sqrt{4\cdot 2}} = \frac{2}{\sqrt{8}} \approx .707$$

$$k_{13} = \frac{1}{\sqrt{2\cdot 3}} = \frac{1}{\sqrt{6}} \approx .4$$

$$k_{23} = \frac{3}{\sqrt{4\cdot 3}} = \frac{3}{\sqrt{12}} = \frac{3}{2\sqrt{3}} \approx .87$$



L1	A	B	2
L2	C	B	3
k12	L1	L2	.4

Coupling coefficient

$$k \triangleq \frac{M_{12}}{\sqrt{L_1 L_2}}$$

Perfect coupling: $k = 1$

Real circuits: $0 < k < 1$

$$k_{12} = \frac{1}{\sqrt{2\cdot 3}} = \frac{1}{\sqrt{6}} \approx .4$$

```
Vin
R1
Rout
L1    2    3    2
L2    3    0    4
k12   L1   L2   .707
L3    3    4    3
k13   L1   L3   .4
k23   L2   L3   .87
.AC
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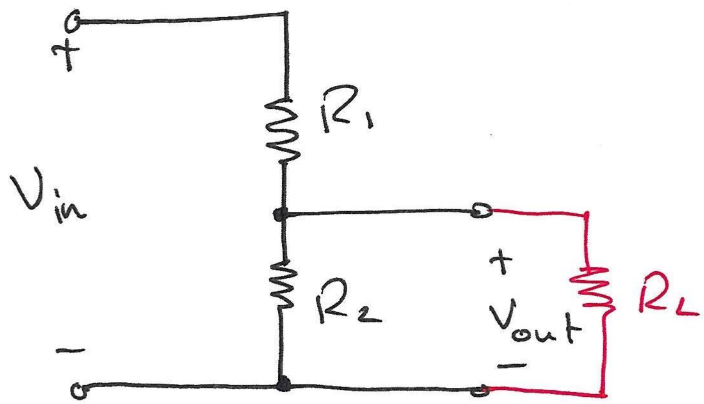
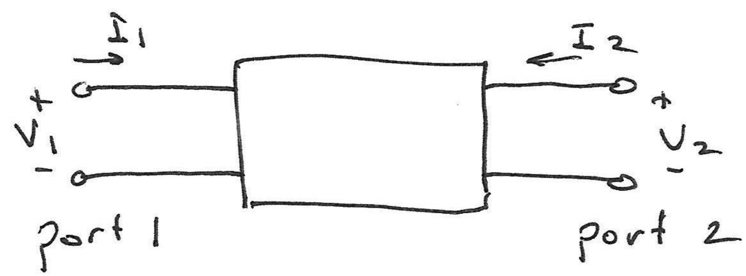


$$\boxed{H = \frac{V_{out}}{V_{in}} = \frac{R_2}{R_1 + R_2}}$$

$$H_{new} = \frac{V_{out}}{V_{in}} = \frac{R_2 \| R_L}{R_1 + R_2 \| R_L}$$

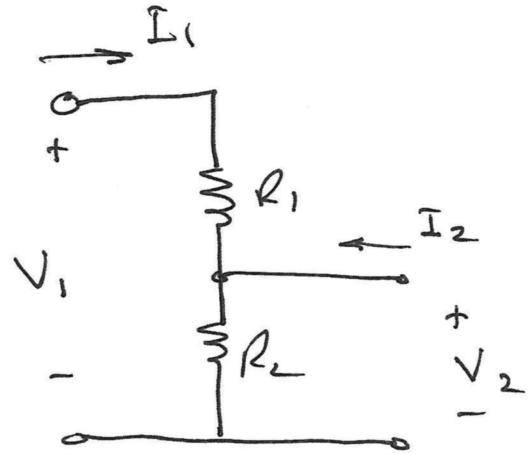
Two-port network description:



Impedance Parameters:

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$



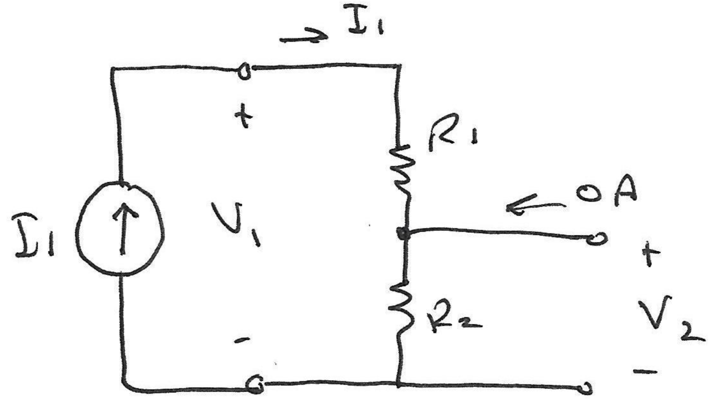
Determine z_{11}, z_{12}, z_{21}, z_{22}

where

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

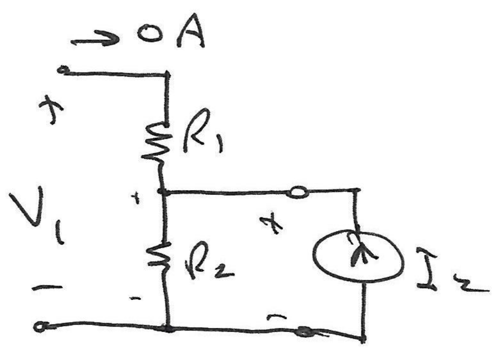
If $I_2 = 0$:



$$V_1 = z_{11} I_1$$
$$= (R_1 + R_2) I_1$$
$$V_2 = R_2 I_1$$

$$z_{11} = R_1 + R_2$$
$$z_{21} = R_2$$

If $I_1 = 0$:



$$V_1 = z_{12} I_2 =$$

$$V_2 = z_{22} I_2 = \underbrace{R_2}_{z_{22}} I_2$$